A Course In Approximation Theory Graduate Studies In Mathematics

This book was originally compiled for a course I taught at the University of Rochester in the fall of 1991, and is intended to give advanced graduate students in statistics an introduction to Edgeworth and saddlepoint approximations, and related techniques. Many other authors have also written monographs on this subject, and so this work is narrowly focused on two areas not recently discussed in theoretical text books. These areas are, first, a rigorous consideration of Edgeworth and saddlepoint expansion limit theorems, and second, a survey of the more recent developments in the field. In presenting expansion limit theorems I have drawn heavily on the books of Daniels, and a review paper by Reid (1988). Throughout this book I have tried to maintain consistent notation and to present them in such a way as to make a few theoretical results useful in as many contexts as possible. This was not only in order to present as many results with as few proofs as possible, but more importantly to show the interconnections between the various facets of asymptotic theory. Special attention is paid to regularity conditions. The reasons they are needed and the parts they play in the proofs are both highlighted.

* Exciting exposition integrates history, philosophy, and mathematics * Combines a mathematical analysis of approximation theory with an engaging discussion of the differing philosophical underpinnings behind its development * Appendices containing biographical data on numerous eminent mathematicians, explanations of Russian nomenclature and academic degrees, and an excellent index round out the presentation.

In this book, we have attempted to explain a variety of different techniques and ideas which have contributed to this subject in its course of successive refinements during the last 25 years. There are other books and surveys reviewing the ideas from the perspective of either potential theory or orthogonal polynomials. The main thrust of this book is to introduce the subject from an approximation theory point of view. Thus, the main motivation is to study analogues of results from classical trigonometric approximation theory, introducing other ideas as needed. It is not our objective to survey the most recent results, but merely to introduce to the readers the thought processes and ideas as they are developed. This book is intended to be self-contained, although the reader is expected to be familiar with rudimentary real and complex analysis. It will also help to have studied elementary trigonometric approximation theory, and have some exposure to orthogonal polynomials.

This is the first systematic study of best approximation theory in inner product spaces and, in particular, in Hilbert space. Geometric considerations play a prominent role in developing and understanding the theory. The only prerequisites for reading the book is some knowledge of advanced calculus and linear algebra.

A Course in Approximation TheoryAmerican Mathematical Soc. In Fourier Analysis and Approximation of Functions basics of classical Fourier Analysis are given as well as those of approximation by polynomials, splines and entire functions of exponential type. In Chapter 1 which has an introductory nature, theorems on convergence, in that or another sense, of integral operators are given. In Chapter 2 basic properties of simple and multiple Fourier series are discussed, while in Chapter 3 those of Fourier integrals are studied. The first three chapters as well as partially Chapter 4 and classical Wiener, Bochner, Bernstein, Khintchin, and Beurling theorems in Chapter 6 might be interesting and available to all familiar with fundamentals of integration theory and elements of Complex Analysis and Operator Theory. Applied mathematicians interested in harmonic analysis and/or numerical methods based on ideas of Approximation Theory are among them. In Chapters 6-11 very recent results are sometimes given in certain directions. Many of these results have never appeared as a book or certain consistent part of a book and can be found only in periodic; looking for them in numerous journals might be quite onerous, thus this book may work as a reference source. The methods used in the book are those of classical analysis, Fourier Analysis in finite-dimensional Euclidean space Diophantine Analysis, and random choice. Approximation Theorems of Mathematical Statistics This convenient paperback edition makes a seminal text in statistics accessible to a new generation of students and practitioners. Approximation Theorems of Mathematical Statistics covers a broad range of limit theorems useful in mathematical statistics, along with methods of proof and techniques of application. The manipulation of "probability" theorems to obtain "statistical" theorems is emphasized. Besides a knowledge of these basic statistical theorems, this lucid introduction to the subject imparts an appreciation of the instrumental role of probability theory. The book makes accessible to students and practicing professionals in statistics, general mathematics, operations research, and engineering the essentials of: * The tools and foundations that are basic to asymptotic theory in statistics * The asymptotics of statistics computed from a sample, including transformations of vectors of more basic statistics, with emphasis on asymptotic distribution theory and strong convergence * Important special classes of statistics, such as maximum likelihood estimates and other asymptotic efficient procedures; W. Hoeffding's U-statistics and R. von Mises's "differentiable statistical functions" * Statistical tests obtained as solutions of equations ("M-estimates"), linear functions of order statistics ("L-statistics"), and rank statistics ("R-statistics") * Use of influence curves * Approaches toward asymptotic relative efficiency of statistical test procedures * Exciting exposition integrates history, philosophy, and mathematics * Combines a mathematical analysis of approximation theory with an engaging discussion of the differing philosophical underpinnings behind its development * Appendices containing biographical data on numerous eminent mathematicians, explanations of Russian nomenclature and academic degrees, and an excellent index round out the presentation.

The field of approximation theory has become so vast that it intersects with every other branch of analysis and plays an increasingly important role in applications in the applied sciences and engineering. Fundamentals of Approximation Theory presents a systematic, in-depth treatment of some basic topics in approximation theory designed to emphasize the rich connections of the subject with other areas of study. With an approach that moves smoothly from the very concrete to more and more abstract levels, this text provides an outstanding blend of classical and abstract topics. The first five chapters present the core of information that readers need to begin research in this domain. The final three chapters the authors devote to special topics-splined functions, orthogonal polynomials, and best approximation in normed linear spaces-illustrate how the framework of approximation theory lends itself to applications in other contexts and expose readers to the use of complex analytic methods in approximation theory. Each chapter contains problems of varying difficulty, including some written by students from contemporary research. Perfect for an introductory graduate-level class, Fundamentals of Approximation Theory also contains enough advanced material to serve more specialized courses at the doctoral level and to interest scientists and engineers.

This book is intended to be used as a textbook for graduate students studying theoretical computer science. It can also be used as a reference book for researchers in the area of design and analysis of approximation algorithms. Design and Analysis of Approximation Algorithms is a graduate course in theoretical computer science taught widely in the universities, both in the United States and abroad. There are, however, very few textbooks available for this course. Among those available in the market, most books follow a problem-oriented format; that is, they collected many important combinatorial optimization problems and their approximation algorithms, and organized them based on the types, or applications, of problems, such as geometric-type problems, algebraic-type problems, etc. Such arrangement of materials is perhaps convenient for a researcher to look for the problems and algorithms related to his/her work, but is difficult for a student to capture the ideas underlying the various algorithms. In the new book proposed here, we follow a more structured, technique-oriented presentation. We organize approximation algorithms into different chapters, based on the design techniques for the algorithms, so that the reader can study approximation algorithms of the
same nature together. It helps the reader to better understand the design and analysis techniques for approximation algorithms, and also helps the teacher to present the ideas and techniques of approximation algorithms in a more unified way. Covering the basic techniques used in the latest research work, the author consolidates progress made so far, including some very recent and promising results, and conveys the beauty and excitement of work in the field. He gives clear, lucid explanations of key results and ideas, with intuitive proofs, and provides critical examples and numerous illustrations to help elucidate the algorithms. Many of the results presented have been simplified and new insights provided. Of interest to theoretical computer scientists, operations researchers, and discrete mathematicians.

This book presents numerical and other approximation techniques for solving various types of mathematical problems that cannot be solved analytically. In addition to well known methods, it contains some non-standard approximation techniques that are now formally collected as well as original methods developed by the author that do not appear in the literature. This book contains an extensive treatise of approximation solutions to various types of integral equations, a topic that is not often discussed in detail. There are detailed analyses of ordinary and partial differential equations and descriptions of methods for estimating the values of integrals that are presented in a level of detail that will suggest techniques that will be useful for developing methods for approximating solutions to problems outside of this text. The book is intended for researchers who must approximate solutions to problems that cannot be solved analytically. It is also appropriate for students taking courses in numerical approximation techniques.

Distills key concepts from linear algebra, geometry, matrices, calculus, optimization, probability and statistics that are used in machine learning.

A self-contained introduction for non-specialists, or a reference work for experts, on the area of approximation theory concerned with exact constants.

The goal of learning theory is to approximate a function from sample values. To attain this goal learning theory draws on a variety of diverse subjects, specifically statistics, approximation theory, and algorithmics. Ideas from all these areas blended to form a subject whose many successful applications have triggered a rapid growth during the last two decades. This is the first book to give a general overview of the theoretical foundations of the subject emphasizing the approximation theory, while still giving a balanced overview. It is based on courses taught by the authors, and is reasonably self-contained so will appeal to a broad spectrum of researchers in learning theory and adjacent fields. It will also serve as an introduction for graduate students and others entering the field, who wish to see how the problems raised in learning theory relate to other disciplines.

This book presents a wide range of well-known and less common methods used for estimating the accuracy of probabilistic approximations, including the Esseen type inversion formulas, the Stein method as well as the methods of convolutions and triangle function. Emphasising the correct usage of the methods presented, each step required for the proofs is examined in detail. As a result, this textbook provides valuable tools for proving approximation theorems. While Approximation Methods in Probability Theory will appeal to everyone interested in limit theorems of probability theory, the book is particularly aimed at graduate students who have completed a standard intermediate course in probability theory. Furthermore, experienced researchers wanting to enlarge their toolkit will also find this book useful.

The approximation of a continuous function by either an algebraic polynomial, a trigonometric polynomial, or a spline, is an important issue in application areas like computer-aided geometric design and signal analysis. This book is an introduction to the mathematical analysis of such approximation, and, with the prerequisites of only calculus and linear algebra, the material is targeted at senior undergraduate level, with a treatment that is both rigorous and self-contained.

The topics include polynomial interpolation; Bernstein polynomials and the Weierstrass theorem; best approximations in the general setting of normed linear spaces and inner product spaces; best uniform polynomial approximation; orthogonal polynomials; Newton-Cotes, Gauss and Clenshaw-Curtis quadrature; the Euler-Maclaurin formula; approximation of periodic functions; the uniform convergence of Fourier series; spline approximation, with an extensive treatment of local spline interpolation, and its application in quadrature. Exercises are provided at the end of each chapter.

The papers in this book, first presented at a 1986 AMS Short Course, give a brief introduction to approximation theory and some of its current areas of active research, both theoretical and applied. The first lecture describes and illustrates the basic concerns of the field. Topics highlighted in the other lectures include the following: approximation in the complex domain, $N$-width, optimal recovery, interpolation, algorithms for approximation, and splines, with a strong emphasis on a multivariate setting for the last three topics. The book is aimed at mathematicians interested in an introduction to areas of current research and to engineers and scientists interested in exploring the field for possible applications to their own fields. The book is best understood by those with a standard first graduate course in real and complex analysis, but some of the presentations are accessible with the minimal requirements of advanced calculus and linear algebra.

Many practical applications require the reconstruction of a multivariate function from discrete, unstructured data. This book gives a self-contained, complete introduction into this subject. It concentrates on truly meshless methods such as radial basis functions, moving least squares, and partitions of unity. The book starts with an overview on typical applications of scattered data approximation, coming from surface reconstruction, fluid-structure interaction, and the numerical solution of partial differential equations. It then leads the reader from basic properties to the current state of research, addressing all important issues, such as existence, uniqueness, approximation properties, numerical stability, and efficient implementation. Each chapter ends with a section giving information on the historical background and hints for further reading. Complete proofs are included, making this perfectly suited for graduate courses on multivariate approximation and it can be used to support courses in computer-aided geometric design, and meshless methods for partial differential equations.

This book presents a unified theory of the Finite Element Method and the Boundary Element Method for a numerical solution of second order elliptic boundary value problems. This includes the solvability, stability, and error analysis as well as efficient methods to solve the resulting linear systems. Applications are the potential equation, the system of linear
elastostatics and the Stokes system. While there are textbooks on the finite element method, this is one of the first books on Theory of Boundary Element Methods. It is suitable for self study and exercises are included.


A rigorous mathematical treatment of the technique for studying the properties of an experimental situation. These are the Proceedings of the NATO Advanced Study Institute on Approximation Theory, Spline Functions and Applications held in the Hotel villa del Mare, Maratea, Italy between April 28,1991 and May 9, 1991. The principal aim of the Advanced Study Institute, as reflected in these Proceedings, was to bring together recent and up-to-date developments of the subject, and to give directions for future research. Amongst the main topics covered during this Advanced Study Institute is the subject of uni variate and multivariate wavelet decomposition over spline spaces. This is a relatively new area in approximation theory and an increasingly important subject. The work involves key techniques in approximation theory cardinal splines, B-splines, Euler-Frobenius polynomials, spline spaces with non-uniform knot sequences. A number of scientific applications are also highlighted, most notably applications to signal processing and digital image processing. Developments in the area of approximation of functions examined in the course of our discussions include approximation of periodic phenomena over irregular node distributions, scattered data interpolation, Pade approximants in one and several variables, approximation properties of weighted Chebyshev polynomials, minimax approximations, and the Strang Fix conditions and their relation to radial functions. I express my sincere thanks to the members of the Advisory Committee, Professors B. Beauzamy, E. W. Cheney, J. Meinguet, D. Roux, and G. M. Phillips. My sincere appreciation and thanks go to A. Carbone, E. DePascale, R. Charron, and B. This textbook offers an accessible introduction to the theory and numerics of approximation methods, combining classical topics of approximation with recent advances in mathematical signal processing, and adopting a constructive approach, in which the development of numerical algorithms for data analysis plays an important role. The following topics are covered: * least-squares approximation and regularization methods * interpolation by algebraic and trigonometric polynomials * basic results on best approximations * Euclidean approximation * Chebyshev approximation * asymptotic concepts: error estimates and convergence rates * signal approximation by Fourier and wavelet methods * kernel-based multivariate approximation * approximation methods in computerized tomography Providing numerous supporting examples, graphical illustrations, and carefully selected exercises, this textbook is suitable for introductory courses, seminars, and distance learning programs on approximation for undergraduate students.

In addition to coverage of univariate interpolation and approximation, the text includes material on multivariate interpolation and multivariate numerical integration, a generalization of the Bernstein polynomials that has not previously appeared in book form, and a greater coverage of Peano kernel theory than is found in most textbooks. There are many worked examples and each section ends with a number of carefully selected problems that extend the student's understanding of the text. The author is well known for his clarity of writing and his many contributions as a researcher in approximation theory.

Approximation Theory and Approximation Practice, Extended Edition differs fundamentally from other works on approximation theory in a number of ways: its emphasis is on topics close to numerical algorithms; concepts are illustrated with Chebfun; and each chapter is a PUBLISHable MATLAB M-file, available online. The book centers on theorems and methods for practical needs. Because the book is based on a course of lectures to third-year undergraduates in mathematics at Cambridge University, sufficient attention is given to theory to make it highly suitable as a mathematical textbook at undergraduate or postgraduate level.

This is a textbook on classical polynomial and rational approximation theory for the twenty-first century. Aimed at advanced undergraduates and graduate students across all of applied mathematics, it uses MATLAB to teach the field's most important ideas and results. Approximation Theory and Approximation Practice, Extended Edition differs fundamentally from other works on approximation theory in a number of ways: its emphasis is on topics close to numerical algorithms; concepts are illustrated with Chebfun; and each chapter is a PUBLISHable MATLAB M-file, available online. The book centers on theorems and methods for analytic functions, which appear so often in applications, rather than on functions at the edge of discontinuity with their seductive theoretical challenges. Original sources are cited rather than textbooks, and each item in the bibliography is accompanied by an editorial comment. In addition, each chapter has a collection of exercises, which span a wide range from mathematical theory to Chebfun-based numerical experimentation. This textbook is appropriate for advanced undergraduate or graduate students who have an understanding of numerical analysis and complex analysis. It is also appropriate for seasoned mathematicians who use MATLAB.

Many physical, chemical, biomedical, and technical processes can be described by partial differential equations or dynamical systems. In spite of increasing computational capacities, many problems are of such high complexity that they are solvable only with severe simplifications, and the design of efficient numerical schemes remains a central research challenge. This book presents a tutorial introduction to recent developments in mathematical methods for model reduction and approximation of complex systems. Model Reduction and Approximation: Theory and Algorithms contains three parts that cover (I) sampling-based methods, such as the reduced basis method and proper orthogonal decomposition, (II) approximation of high-dimensional problems by low-rank tensor techniques, and (III) system-theoretic methods, such as balanced truncation, interpolatory methods, and the Loewner framework. It is tutorial in nature, giving an accessible introduction to state-of-the-art model reduction and approximation methods. It also covers a wide range of methods drawn from typically distinct communities (sampling based, tensor based, system-theoretic). ?? This book is intended for researchers interested in model reduction and approximation, particularly graduate students and young researchers.

This unique textbook emphasizes the commonalities between splines and fractals in interpolation and approximation theory, with particular emphasis on fractal functions and fractal surfaces. It presents the classical theory of splines and their properties, and also gives an introduction to the burgeoning new theory of superfractals and superfractal functions.
Engineering systems operate through actuators, most of which will exhibit phenomena such as saturation or zones of no operation, commonly known as dead zones. These are examples of piecewise-affine characteristics, and they can have a considerable impact on the stability and performance of engineering systems. This book targets controller design for piecewise affine systems, fulfilling both stability and performance requirements. The authors present a unified computational methodology for the analysis and synthesis of piecewise affine controllers, taking an approach that is capable of handling sliding modes, sampled-data, and networked systems. They introduce algorithms that will be applicable to nonlinear systems approximated by piecewise affine systems, and they feature several examples from areas such as switching electronic circuits, autonomous vehicles, neural networks, and aerospace applications. Piecewise Affine Control: Continuous-Time, Sampled-Data, and Networked Systems is intended for graduate students, advanced senior undergraduate students, and researchers in academia and industry. It is also appropriate for engineers working on applications where switched linear and affine models are important.

Adapted from a series of lectures given by the authors, this monograph focuses on radial basis functions (RBFs), a powerful numerical methodology for solving PDEs to high accuracy in any number of dimensions. This method applies to problems across a wide range of PDEs arising in fluid mechanics, wave motions, astro- and geosciences, mathematical biology, and other areas and has lately been shown to compete successfully against the very best previous approaches on some large benchmark problems. Using examples and heuristic explanations to create a practical and intuitive perspective, the authors address how, when, and why RBF-based methods work. The authors trace the algorithmic evolution of RBFs, starting with brief introductions to finite difference (FD) and pseudospectral (PS) methods and following a logical progression to global RBFs and then to RBF-generated FD (RBF-FD) methods. The RBF-FD method, conceived in 2000, has proven to be a leading candidate for numerical simulations in an increasingly wide range of applications, including seismic exploration for oil and gas, weather and climate modeling, and electromagnetics, among others. This is the first survey in book format of the RBF-FD methodology and is suitable as the text for a one-semester first-year graduate class.

Discrete optimization problems are everywhere, from traditional operations research planning (scheduling, facility location and network design); to computer science databases; to advertising issues in viral marketing. Yet most such problems are NP-hard; unless P = NP, there are no efficient algorithms to find optimal solutions. This book shows how to design approximation algorithms: efficient algorithms that find provably near-optimal solutions. The book is organized around central algorithmic techniques for designing approximation algorithms, including greedy and local search algorithms, dynamic programming, linear and semidefinite programming, and randomization. Each chapter in the first section is devoted to a single algorithmic technique applied to several different problems, with more sophisticated treatment in the second section. The book also covers methods for proving that optimization problems are hard to approximate. Designed as a textbook for graduate-level algorithm courses, it will also serve as a reference for researchers interested in the heuristic solution of discrete optimization problems.

Calculus from Approximation to Theory takes a fresh and innovative look at the teaching and learning of calculus. One way to describe calculus might be to say it is a suite of techniques that approximate curved things by flat things and through a limiting process applied to those approximations arrive at an exact answer. Standard approaches to calculus focus on that limiting process as the heart of the matter. This text places its emphasis on the approximating processes and thus illuminates the motivating ideas and makes clearer the scientific usefulness, indeed centrality, of the subject while paying careful attention to the theoretical foundations. Limits are defined in terms of sequences, the derivative is defined from the best affine approximation, and greater attention than usual is paid to numerical techniques and the order of an approximation. Access to modern computational tools is presumed throughout and the use of these tools is woven seamlessly into the exposition and problems. All of the central topics of a yearlong calculus course are covered, with the addition of treatment of difference equations, a chapter on the complex plane as the arena for motion in two dimensions, and a much more thorough and modern treatment of differential equations than is standard. Dan Sloughter is Emeritus Professor of Mathematics at Furman University with interests in probability, statistics, and the philosophy of mathematics and statistics. He has been involved in efforts to reform calculus instruction for decades and has published widely on that topic. This book, one of the results of that work, is very well suited for a yearlong introduction to calculus that focuses on ideas over techniques.

This textbook is designed for graduate students in mathematics, physics, engineering, and computer science. Its purpose is to guide the reader in exploring contemporary approximation theory. The emphasis is on multi-variable approximation theory, i.e., the approximation of functions in several variables, as opposed to the classical theory of functions in one variable. Most of the topics in the book, heretofore accessible only through research papers, are treated here from the basics to the currently active research, often motivated by practical problems arising in diverse applications such as science, engineering, geophysics, and business and economics. Among these topics are projections, interpolation paradigms, positive definite functions, interpolation theorems of Schoenberg and Micchelli, tomography, artificial neural networks, wavelets, thin-plate splines, box splines, ridge functions, and convolutions. An important and valuable feature of the book is the bibliography of almost 600 items directing the reader to important books and research papers. There are 438 problems and exercises scattered through the book allowing the student reader to get a better understanding of the subject.

The series is devoted to the publication of monographs and high-level textbooks in mathematics, mathematical methods and their applications. Apart from covering important areas of current interest, a major aim is to make topics of an interdisciplinary nature accessible to the non-specialist. The works in this series are addressed to advanced students and researchers in mathematics and theoretical physics. In addition, it can serve as a guide for lectures and seminars on a graduate level. The series de Gruyter Studies in Mathematics was founded ca. 30 years ago by the late Professor Heinz Bauer and Professor Peter Gabriel with the aim to establish a series of monographs and textbooks of high standard, written by scholars with an international reputation presenting current fields of research in pure and applied mathematics. While the editorial board of the Studies has changed with the years, the aspirations of the Studies are unchanged. In times of rapid growth of mathematical knowledge carefully written monographs and textbooks written by experts are needed more than ever, not least to pave the way for the next generation of mathematicians. In this sense the editorial board and the publisher of the Studies are devoted to continue the Studies as a service to the mathematical community. Please submit any book proposals to Niels Jacob.

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